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Abstract. Langmuir wave turbulence generated by a beam-plasma interaction has been studied since the early days of plasma physics research. Despite a long history of investigation on this subject, among the outstanding issues is the generation of harmonic Langmuir waves observed in both laboratory and computer-simulated experiments. However, the phenomenon has not been adequately explained in terms of theory, nor has it been fully characterized by means of numerical simulations. In this paper, a theory of harmonic Langmuir wave generation is put forth and tested against the Vlasov simulation results. It is found that the harmonic Langmuir mode spectra exhibit quasi power-law feature implying a multi-scale structure in both frequency and wave number space spanning several orders of magnitude.

1 Introduction

It is often said that the turbulence is one of the unsolved problems of classical physics. By "turbulence", it is meant here as the turbulence in neutral fluids governed by Navier-Stokes equation. Most of the studies on turbulence is in this context (Kolmogorov, 1941; McComb, 1990). The turbulence in plasmas is even less well-understood. For low-frequency turbulence in magnetized plasmas, at least the macroscopic magnetohydrodynamic (MHD) equation, which is similar to Navier-Stokes equation in neutral fluids is applicable, and as such, many conceptual and theoretical tools originally developed for the fluid turbulence can be employed (e.g. Iroshnikov, 1964; Kraichnan, 1965). However, for high-frequency plasma turbulence where microscopic wave-particle interaction becomes important, the situation is quite different from the low-frequency MHD or fluid turbulence.

Many physical systems exhibit turbulent behavior which can be characterized by quasi scale-free structure associated with the fluctuations, as exemplified by the power-law spectral distribution. Modern research on turbulence which began with the pioneering work of Kolmogorov in the 1940s (Kolmogorov, 1941), stems largely from neutral fluid turbulence, and terminologies such as inertial and/or dissipation range behaviors, etc., often arise out of the context of the Navier-Stokes equation. However, the plasma exhibits behaviors associated with fluctuations which are superficially similar to the fluid turbulence. Hence, the notions developed in the context of the fluid turbulence are sometimes indiscriminately applied to plasmas on the basis of gross morphological features, even though one does not really understand the underlying dynamics.

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2 A brief history of Langmuir turbulence

Serious investigations of (high-frequency) plasma turbulence can be said to have begun with the works of the scientists largely from the former Soviet Union in the early 1960s (e.g. standard monographs on the subject are, Kadomtsev, 1965; Sitenko, 1967, 1982; Vedenov, 1968; Sagdeev and Galeev, 1969; Tsytovich, 1970, 1977a, b; Davdison, 1972; Kaplan and Tsytovich, 1973; Hasegawa, 1975; Akhiezer et al., 1975; Melrose, 1980). Efforts by these pioneers, which came to be known as the plasma weak turbulence theory, continued on through the 1970s and 1980s. It should be noted that although the weak turbulence formalism is quite general, and in principle it can be applied to a wide variety of problems, in practice however, it is almost exclusively applied to the bump-on-tail (or weak beam-plasma) instability problem, which is one of the simplest plasma instabilities. Hence, the Langmuir turbulence problem became the testbed for various plasma turbulence theories.

During the trail-blazing days, scientists in the West were largely following the Soviet scientists' lead, but a few made important contributions of their own. For instance, Dupree (1966, 1972), Weinstock (1969), and others, suggested the renormalized turbulence theories, which is an effort to go beyond the weak turbulence perturbation scheme and to take the higher-order terms into account. (In the early days, the renormalized kinetic theories were called "the strong turbulence" theories, but we use the term "renormalized" to distinguish them from the later theory of the same name by Zakharov). However, findings from these sophisticated theories and earlier weak turbulence theories were directly challenged by results from numerical simulations, which often showed coherent nonlinear effects (such as particle trapping by large-amplitude waves) playing an important, if not the dominant, role (Dawson and Shanny, 1968; Morse and Nielson, 1969).

Compounding the inability of the weak or renormalized turbulence theories to account for the coherent nonlinear dynamics was the fact that these theories simply could not produce quasi scale-free power-law type of spectrum associated with the Langmuir turbulence. The major reason is the dominance of linear physics in plasmas, unlike the fluids. In neutral fluids, the Navier-Stokes equation can be Fourier transformed into

$$\left(\frac{\partial}{\partial t}+\nu k^2\right) v_i(\boldsymbol{k}) = M_{ijm}(\boldsymbol{k}) \int d\boldsymbol{k}'\; v_j(\boldsymbol{k}')\, v_m(\boldsymbol{k}-\boldsymbol{k}'),$$

where $v_i(\boldsymbol{k})$ is the Fourier component of the perturbed fluid velocity vector, ν is the fluid viscosity, and

$$M_{ijm}(\boldsymbol{k}) = -(i/2)\, P_{ijm}(\boldsymbol{k}),$$
$$P_{ijm}(\boldsymbol{k}) = k_m\, P_{ij}(\boldsymbol{k}) + k_j\, P_{im}(\boldsymbol{k}),$$
$$P_{ij}(\boldsymbol{k}) = \delta_{ij} - k_i k_j / k^2,$$

is the coupling factor. In fluids, ν is a small parameter, and many problems can be discussed by entirely ignoring the linear dissipation term, νk^2. In Vlasov plasmas, on the other hand, the Fourier component of the perturbed distribution function obeys an equation of the form

$$\left(\frac{\partial}{\partial t} - i(\omega_{\boldsymbol{k}} - \boldsymbol{k}\cdot\boldsymbol{v}) + \gamma_{\boldsymbol{k}}\right) f_{\boldsymbol{k}}(\boldsymbol{v}) = \frac{ie}{m}\,\phi_{\boldsymbol{k}}\,\boldsymbol{k}\cdot\frac{\partial f_0(\boldsymbol{v})}{\partial \boldsymbol{v}}$$
$$+\frac{ie}{m}\int d\boldsymbol{k}'\;\boldsymbol{k}'\cdot\frac{\partial}{\partial \boldsymbol{v}}\Big(\phi_{\boldsymbol{k}'}\, f_{\boldsymbol{k}-\boldsymbol{k}'} - \langle \phi_{\boldsymbol{k}'}\, f_{\boldsymbol{k}-\boldsymbol{k}'}\rangle\Big),$$

where $\phi_{\boldsymbol{k}}$ is the perturbed electrostatic field which must be determined self-consistently from Poisson's equation, and $\omega = \omega_{\boldsymbol{k}} + i\gamma_{\boldsymbol{k}}$ is the complex wave dispersion relation, $\gamma_{\boldsymbol{k}}$ being the Landau damping factor. Unlike the fluids, the dominant term in the plasma is the linear term, especially, the Landau damping factor.

Owing to the insignificance of dissipation in fluids, the dominant-scale eddy can freely break into smaller and smaller eddies, thus creating the well-known stationary Kolmogorov $k^{-5/3}$ scaling in the inertial range. In plasmas, however, the cascading of the bump-on-tail generated Langmuir waves to shorter wavelength modes is prevented by the strong Landau damping. On the other hand, the upside of the dominance of the linear physics is that it makes the perturbation expansion (i.e. weak turbulence theory) to become quite valid. For Navier-Stokes turbulence, in contrast, as seen above, there is no dominant linear term around which a suitable perturbation expansion can be employed. As a result, the fluid turbulence problem necessitates some sort of renormalization or another at the outset.

In 1972, Zakharov proposed a semi-phenomenological theory of plasma turbulence, which came to be known as the strong turbulence theory. In his theory, the collapse of intense Langmuir wave packet plays the prominent role. The strong turbulence theory ignores the wave-particle effect, and is a macroscopic theory, but the beauty of the theory is that it predicts a certain power-law scaling resulting from the collapse of the wave packets (Zakharov, 1972; Galeev et al., 1975). Because of this, the attention in the community gradually shifted to the Zakharov's strong turbulence theory (see the reviews by Goldman, 1984; Robinson, 1997). However, as mentioned already, Zakharov theory ignores the microscopic wave-particle effect (e.g. Landau damping), and to this date the theory remains controversial, as various numerical simulations and experiments to confirm the theory are inconclusive (Robinson and Newman, 1990; Vyacheslavov et al., 2002; Erofeev, 2002).

Here, it should be noted that Zakharov-type of highly nonlinear and coherent theory of Langmuir waves plays a prominent role in high power radio wave experiment in the ionosphere, and such a theory enjoys many successes there (Dubois et al., 2001, Cheung et al., 2001). However, the present discussion is concerned with incoherent turbulence problem where the nonlinearity is relatively weak. For such a weakly turbulent situation, the Zakharov theory remains controversial (Erofeev, 2002).

By the mid 1980s, the community began to realize that the notion of plasma turbulence as nonlinear interactions of Fourier eigenmodes (weak turbulence picture) is insufficient, and/or that semi-phenomenological approach of Zakharov (strong turbulence picture) remains inconclusive. Moreover, the issue of coherent nonlinear effects remained wide open. However, various attempts were made to improve upon the early renormalized kinetic theories to bring the particle trapping effects (i.e. coherent nonlinear physics) into the picture.

As a result, Adam et al. (1979) and Laval and Pesme (1983) claimed that the so-called turbulent trapping effect invalidates the quasilinear/weak turbulence theory at the lowest order. However, efforts to confirm their theoretical prediction either by means of numerical simulations or by experiments did not produce positive results (Theilhaber et al., 1987; Tsunoda et al., 1987). Approximately a decade passed when Liang and Diamond (1993) performed a correct analysis of renormalized kinetic theory to show that the effects predicted by Adam et al. (1979) and Laval and Pesme (1983) were insignificant, thus reestablishing the validity of the original quasilinear/weak turbulence approach. However, the fact still remained that the weak turbulence theory could not account for the dominant coherent nonlinear dynamics observed in early simulations.

In 1990, however, Dum (1990a, b, c) carried out detailed particle-in-cell simulations to show that the dominant particle trapping behavior observed in early simulations were partly owing to the insufficient mode resolution and small system size. He proceeded to demonstrate with his refined simulations that quasilinear/weak turbulence theories are actually quite good for certain parameter regime. Specifically, for a weak and warm beam, the weak turbulence theory provided an acceptable first-order description of the nonlinear behavior of the system. For a more recent discussions on the preponderance of incoherent versus coherent nonlinear effects in beam-plasma interactions, see the discussion by Omura et al. (1996). Notwithstanding these developments, however, the issue of the lack of mechanism in the weak turbulence theory to generate power-law turbulence spectra still remained outstanding.

3 Harmonic Langmuir modes

In the mean time, over the past four decades or so, evidence from laboratory and spaceborne experiments (Apel, 1967, 1969; Malmberg and Wharton, 1969; Mizuno and Tanaka, 1972; Gentle and Lohr, 1973; Mori, 1973; Seidl et al., 1976; Boswell and Kellogg, 1983; Llobet et al., 1985; Kellogg et al., 1986) as well as from numerical computer simulations (Joyce et al., 1971; Goldstein et al., 1978; Pritchett and Dawson, 1983; Klimas, 1983, 1990; Akimoto et al., 1988; Nishikawa and Cairns, 1991; Yin et al., 1998; Viñas et al., 2000; Schriver et al., 2000; Kasaba et al., 2001) accumulated which showed that the real Langmuir turbulence involves the so-called harmonic mode generation, and that the power-law spectrum associated with the Langmuir turbulence involves these harmonic modes. Such a phenomenon cannot be accounted for on the basis of available theories, despite some early efforts (Manheimer, 1971; O'Neil et al., 1971; O'Neil and Winfrey, 1972).

These early theories of harmonic Langmuir modes may be highly relevant to coherent harmonic generation in the case of monoenergetic beams, but they may not be directly relevant to the turbulent generation of the harmonic modes. As a matter of fact, coherent harmonic mode generation is a well-known phenomenon in microwave generation devices (Uhm and Chen, 1993), and the coherent nonlinear theory by Uhm and Chen appear to be very similar to early theories of O'Neil and his colleagues. Generation of harmonic components in turbulent plasmas is another matter which is quite distinct from coherent nonlinear phenomenon.

It seemed that the true picture of plasma turbulence involves the harmonic Langmuir modes, which are nonlinear eigenmodes of turbulent plasmas. In contrast, the traditional plasma turbulence theories assume that the basic modes are linear eigenmodes of a quiescent plasma. Through the excitation of nonlinear eigenmodes, which have no conceptual counterpart in fluid turbulence, the cascading of the primary bump-on-tail mode to shorter wavelength mode seemed to be established without suffering heavy Landau damping, and thus the power-law turbulence spectrum is established. Observations show that the peaks of the harmonic spectra form a steep power-law with an index ~ -5 or -6.

According to the laboratory and simulation results, the harmonic waves are excited at multiples of the plasma frequency, $\omega \sim n\omega_{pe},\ n = 2, 3, 4, \cdots$ (here, $\omega_{pe}^2 = 4\pi\hat{n}e^2/m_e$ is the square of the electron plasma frequency, $\hat{n}$, e, and m_e being the ambient density, unit electric charge, and electron rest mass, respectively), and they all propagate with phase speeds roughly equal to the beam propagation speed, $\omega/k \sim V_0$, where V_0 is the average electron beam speed. This is the reason why these modes are not Landau damped, despite the fact that the wavelengths are much shorter than the primary Langmuir mode.

Theoretical understanding of the harmonic generation in turbulent plasmas is very limited, although the analogous phenomenon involving monoenergetic electron beam is understood much better in terms of forced electrostatic perturbations (Joyce et al., 1971; Klimas, 1983, 1990; Manheimer, 1971; O'Neil et al., 1971; O'Neil and Winfrey, 1972; Uhm and Chen, 1993). Recently, however, a theory of harmonic generation in turbulent plasmas was developed by the author and his colleagues (Yoon, 2000; Gaelzer et al., 2002, 2003; Yoon et al., 2003; Umeda et al., 2003), in which these modes are considered as eigenmodes of nonlinear plasma system. This approach was prompted by recent simulations (Schriver et al., 2000; Kasaba et al., 2001) which show that harmonic modes persist even in late nonlinear phase when the coherent phase space structure is no longer apparent, and when the plasma has entered a stage which can be genuinely characterized by random-phases. Once the coherency is lost, the harmonic modes can no longer be viewed simply as coherent forced electrostatic perturbations.

4 Nonlinear dispersion relation for harmonic Langmuir modes

The detailed derivation of the nonlinear eigenmode solution, i.e. the harmonic mode dispersion relation, is given in the paper by Yoon et al. (2003), and thus shall not be repeated here. Instead, we briefly outline the general procedure. The starting point of the present analysis is the formal nonlinear spectral balance equation, given by Eq. (3) of the paper by Yoon (2000),

$$0 = \left(\frac{i}{2}\frac{\partial\epsilon(\kappa)}{\partial\omega}\frac{\partial}{\partial t} + \epsilon(\kappa)\right) I(\kappa) - 2\int d\kappa' \left[\frac{|\chi^{(2)}(\kappa'|\kappa-\kappa')|^2}{\epsilon^*(\kappa)} I(\kappa')\, I(\kappa-\kappa') - \{\chi^{(2)}(\kappa'|\kappa-\kappa')\}^2 \cdot \left(\frac{I(\kappa-\kappa')}{\epsilon(\kappa')} + \frac{I(\kappa')}{\epsilon(\kappa-\kappa')}\right) I(\kappa) + \bar{\chi}^{(3)}(\kappa'|-\kappa'|\kappa)\, I(\kappa')\, I(\kappa)\right], \tag{1}$$

where $\kappa = (\boldsymbol{k}, \omega)$, $\kappa' = (\boldsymbol{k}', \omega')$, $\kappa - \kappa' = (\boldsymbol{k} - \boldsymbol{k}', \omega - \omega')$, and $\int d\kappa' = \int d\boldsymbol{k}' \int d\omega'$. In Eq. (1),

$$\epsilon(\kappa) = 1 + \chi(\kappa), \quad \chi(\kappa) = \sum_a \frac{\omega_{pa}^2}{k^2} \int d\boldsymbol{v}\, \boldsymbol{k} \cdot \boldsymbol{g}_\kappa\, f_a,$$

$$\chi^{(2)}(\kappa'|\kappa - \kappa') = \frac{-i}{2} \sum_a \frac{e_a}{m_a} \frac{\omega_{pa}^2}{kk'|\boldsymbol{k} - \boldsymbol{k}'|} \int d\boldsymbol{v}\, \{(\boldsymbol{k}' \cdot \boldsymbol{g}_\kappa)\, [(\boldsymbol{k} - \boldsymbol{k}') \cdot \boldsymbol{g}_{\kappa-\kappa'}\, f_a] + (\kappa' \leftrightarrow \kappa - \kappa')\},$$

$$\chi^{(3)}(\kappa'| - \kappa'|\kappa) = \frac{1}{2} \sum_a \frac{e_a^2}{m_a^2} \frac{\omega_{pa}^2}{k^2 k'^2} \int d\boldsymbol{v}\, (\boldsymbol{k}' \cdot \boldsymbol{g}_\kappa)\, [(\boldsymbol{k}' \cdot \boldsymbol{g}_{\kappa-\kappa'})(\boldsymbol{k} \cdot \boldsymbol{g}_\kappa\, f_a) + (\kappa \leftrightarrow -\kappa')],$$

are the various linear and nonlinear plasma response functions, $f_a(\boldsymbol{v})$ is the velocity distribution function for species a [normalized to unity, $\int d\boldsymbol{v}\ f_a(\boldsymbol{v}) = 1$],

$$\boldsymbol{g}_\kappa = \frac{1}{\omega - \boldsymbol{k} \cdot \boldsymbol{v} + i0} \frac{\partial}{\partial \boldsymbol{v}},$$

and the summation $\sum_a$ is over the particle species, with $\omega_{pa}^2 = 4\pi \hat{n} e_a^2 / m_a$ representing the square of the plasma frequency for species $a = e, i$ (e and i stand for the electrons and ions). The quantity $I(\kappa)$ is the phase-average over the square of the wave electric field,

$$I(\kappa) = \left\langle \delta E^2 \right\rangle_{\boldsymbol{k},\omega}.$$

The derivation of Eq. (1) and approximate forms of the various response functions can be found in the paper by Yoon (2000). We simply mention that the desired dispersion equation is obtained from the real part of Eq. (1), while the imaginary part leads to the wave kinetic equation. The total spectral wave intensity, $I(\kappa)$, is given by the sum of individual wave intensity for each normal mode, designated by α,

$$I(\kappa) = \sum_\alpha \sum_{\sigma=\pm1} I_\alpha^\sigma(\boldsymbol{k})\, \delta(\omega - \sigma\omega_{\boldsymbol{k}}^\alpha). \tag{2}$$

Specifically, we shall adopt $\alpha = Ln$ to designate the nth-harmonic Langmuir wave. For $n = 1$ (the fundamental Langmuir mode), the customary Bohm-Gross dispersion relation, $\omega_{\boldsymbol{k}}^{L1} = \omega_{pe}(1 + 3k^2\lambda_{De}^2/2)$, is well-known, where $\lambda_{De}^2 = T_e/(4\pi\hat{n}e^2)$ is the square of the debye length, T_e being the electron temperature.

By inserting Eq. (2) into the real part of Eq. (1), we obtain a nonlinear dispersion equation given by

$$0 = \mathrm{Re}\left(\epsilon(\boldsymbol{k}, \sigma\omega_{\boldsymbol{k}}^{Ln}) - 4 \sum_{\sigma'=\pm1} \int d\boldsymbol{k}'\ T_{\boldsymbol{k},\boldsymbol{k}'}\right) I_{Ln}^\sigma(\boldsymbol{k}), \tag{3}$$

$$T_{\boldsymbol{k},\boldsymbol{k}'} = \frac{|\chi^{(2)}(\boldsymbol{k}', \sigma'\omega_{\boldsymbol{k}'}^{L(n-1)}|\boldsymbol{k} - \boldsymbol{k}', \sigma\omega_{\boldsymbol{k}}^{Ln} - \sigma'\omega_{\boldsymbol{k}'}^{L(n-1)})|^2}{\epsilon(\boldsymbol{k} - \boldsymbol{k}', \sigma\omega_{\boldsymbol{k}}^{Ln} - \sigma'\omega_{\boldsymbol{k}'}^{L(n-1)})}\, I_{L(n-1)}^{\sigma'}(\boldsymbol{k}'),$$

where $n > 2$. After some suitable approximations are made, Eq. (3) can be shown to reduce to

$$1 \approx \frac{n^2}{2\,(n^2 - 1)} \frac{e^2}{m_e^2\, \omega_{pe}^3} \int d\boldsymbol{k}' \frac{A_{\boldsymbol{k},\boldsymbol{k}'}^{(n)}\, I_{L(n-1)}^+(\boldsymbol{k}')}{\omega - \omega_{\boldsymbol{k}'}^{L(n-1)} - \omega_{\boldsymbol{k}-\boldsymbol{k}'}^{L1}},$$

$$A_{\boldsymbol{k},\boldsymbol{k}'}^{(n)} = \left\{\left[(n-1)\, k^2\, \boldsymbol{k}' + n\, k'^2\, \boldsymbol{k}\right] \cdot (\boldsymbol{k} - \boldsymbol{k}')\ + n\,(n-1)\, |\boldsymbol{k} - \boldsymbol{k}'|^2\, (\boldsymbol{k} \cdot \boldsymbol{k}')\right\}^2 \left\{n^4\,(n-1)^4\, k^2\, k'^2\, |\boldsymbol{k} - \boldsymbol{k}'|^2\right\}^{-1}, \tag{4}$$

where $\omega_{\boldsymbol{k}-\boldsymbol{k}'}^{L1}$ is the fundamental Langmuir mode dispersion relation.

The detailed deductive analysis of the above equation leads to the following desired dispersion relation for the nth-harmonic Langmuir mode:

$$\omega_{\boldsymbol{k}}^{Ln} = \omega_{pe}\left(n + \varepsilon_{\boldsymbol{k}}^{(n)} + \frac{3}{2} k^2 \lambda_{De}^2 + \frac{3\theta_{\boldsymbol{k}}^{(n)}}{\varepsilon_{\boldsymbol{k}}^{(n)}} \lambda_{De}^2\right), \tag{5}$$

where

$$\varepsilon_{\boldsymbol{k}}^{(n)} = \frac{n^2}{2\,(n^2 - 1)} \frac{e^2}{m_e^2\, \omega_{pe}^4} \int d^3\boldsymbol{k}'\ A_{\boldsymbol{k},\boldsymbol{k}'}^{(n)}\, I_{L(n-1)}(\boldsymbol{k}'),$$

$$\theta_{\boldsymbol{k}}^{(n)} = \frac{n^2}{2\,(n^2 - 1)} \frac{e^2}{m_e^2\, \omega_{pe}^4} \int d^3\boldsymbol{k}'\ A_{\boldsymbol{k},\boldsymbol{k}'}^{(n)}\, I_{L(n-1)}(\boldsymbol{k}') \left(k'^2 - \boldsymbol{k} \cdot \boldsymbol{k}' + \frac{\theta_{\boldsymbol{k}'}^{(n-1)}}{\varepsilon_{\boldsymbol{k}'}^{(n-1)}}\right), \tag{6}$$

with $\theta_{\boldsymbol{k}}^{(1)} = 0$. For the purpose of illustration, let us consider a specific model for the harmonic Langmuir-wave spectra. As noted already, on the basis of simulations and experiments, the nth-harmonic Langmuir mode (Ln) can be modeled with a spectrum with average wave number located at roughly $nk_0 \approx n\omega_{pe}/V_0$. On the basis of this consideration, we model the one-dimensional harmonic Langmuir mode spectra by

$$I_{Ln}(k) = I_n\,(\pi^{1/2}\,\delta)^{-1}\,e^{-(k-nk_0)^2/\delta^2},$$

where $I_n = \int dk\; I_{Ln}(k)$, and δ represents the spread associated with the spectra. This leads to

$$\frac{\omega_k^{Ln}}{\omega_{pe}} = n + \frac{3}{2}\,k^2\lambda_{De}^2 - \frac{3\,(n-1)}{2}\left(\frac{k}{k_0} - \frac{n}{2}\right)\frac{k_0^2 v_e^2}{\omega_{pe}^2}, \tag{7}$$

where $k_0 \approx \omega_{pe}/V_0$.

To test the idea of the present nonlinear eigen-mode theory of harmonic generation, we have also performed one-dimensional electrostatic Vlasov simulation. The full details of the simulation technique and the in-depth analysis of the results are the focus of the paper by Umeda et al. (2003). The simulation result shown in Fig. 1 corresponds to the intensity of the waves plotted in grayscale format against normalized frequency and wavenumber, ω/ω_{pe} and kV_0/ω_{pe}. The input parameters for the simulation are $n_b/n_0 = 10^{-3}$, $V_0 = 3.5\,v_e$, and $T_e/T_b = 4$. The Fourier transformation is performed over the simulated data in both space and time, and the result is a simulated ω-k dispersion diagram as shown in Fig. 1. We have supperposed the theoretical dispersion relation curves, given by Eq. (7) on top of the numerically generated wave intensity versus ω and k. The result is the comparison between the theory and simulation. The result is an excellent agreement between the simulation result and theory.

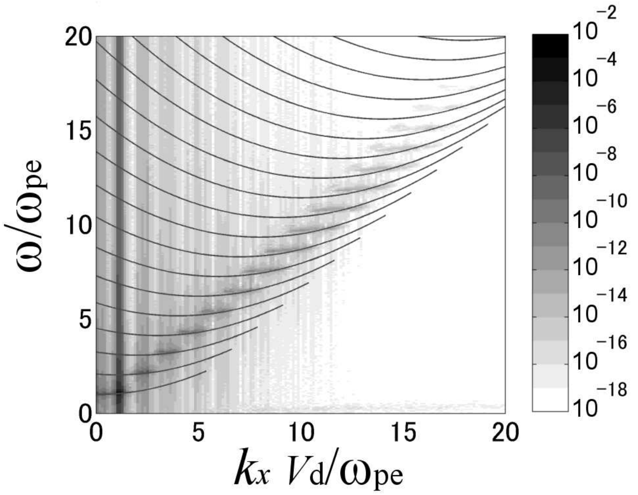

Fig. 1. Simulated dispersion diagram for harmonic Langmuir waves. The intensity of the waves obtained via Vlasov simulation are Fourier analyzed in both space and time, and the result is plotted in grayscale format against normalized frequency and wavenumber, ω/ω_{pe} and kV_0/ω_{pe}. We have superposed the theoretical dispersion relation curves (7).

5 Wave kinetic equation and saturated wave spectrum

Now let us consider the imaginary part of the nonlinear spectral balance Eq. (1). For the fundamental Langmuir mode ($L1$), the complete nonlinear wave kinetic equation, which includes nonlinear decay and induced scattering processes, have already been derived by Yoon (2000) and numerically solved by Ziebell et al. (2001). For the present purpose, however, the nonlinear wave couplings do not play a significant role, and since the time domain of interest is sufficiently short, the effective wave kinetic equations for all harmonic modes are the quasilinear wave kinetic equation,

$$\frac{\partial I_{Ln}^{\sigma}(\boldsymbol{k})}{\partial t} = n^2\pi\sigma\omega_{\boldsymbol{k}}^{Ln}\frac{\omega_{pe}^2}{k^2}\int d\boldsymbol{v}\;\delta(\sigma\omega_{\boldsymbol{k}}^{Ln} - \boldsymbol{k}\cdot\boldsymbol{v})$$
$$\times\,\boldsymbol{k}\cdot\frac{\partial f_e(\boldsymbol{v})}{\partial\boldsymbol{v}}\,I_{Ln}^{\sigma}(\boldsymbol{k}). \tag{8}$$

This equation needs to be closed by the particle kinetic equation for the electrons,

$$\frac{\partial f_e(\boldsymbol{v})}{\partial t} = \frac{\pi^2 e^2}{m_e^2}\frac{\partial}{\partial v_i}\sum_{\sigma=\pm1}\sum_n\int d\boldsymbol{k}\;\frac{k_i\,k_j}{k^2}\,I_{Ln}^{\sigma}(\boldsymbol{k})$$
$$\times\,\delta(\sigma\omega_{\boldsymbol{k}}^{Ln} - \boldsymbol{k}\cdot\boldsymbol{v})\,\frac{\partial f_e(\boldsymbol{v})}{\partial v_j}. \tag{9}$$

Equation (8) has the same structure as the conventional quasilinear wave kinetic equation, except for the overall coefficient proportional to n^2. Therefore, even without solving this equation, one can readily see that the harmonic modes will start to grow in the linear regime, provided that a minimum level of spectral intensity exists for these modes.

The initial electron distribution function is given by a Maxwellian (thermal) core plus an energetic beam component, while the ions are treated as quasi-stationary. We have numerically solved the complete set of wave and particle kinetic equations, Eqs. (8) and (9) in one-dimensional limit. In the present scheme, we employ the following *ad hoc* procedure to define the initial state for all eigenmodes: For a given n-th harmonic, the small level of initial spectrum is modeled by a gaussian form,

$$I_{Ln}(k) = \frac{In}{\sqrt{\pi}\,\Delta}\,\exp\!\left(-\frac{(k-nk_{L1})^2}{\Delta^2}\right), \tag{10}$$

where k_{L1} is the normalized wave number associated with the primary $L1$ mode. This choice is guided by the linear growth property which dictates that the n-th harmonic mode should grow in the vicinity of $k \sim nk_{L1}$, with some spread in wave number, characterized by Δ. The precise functional

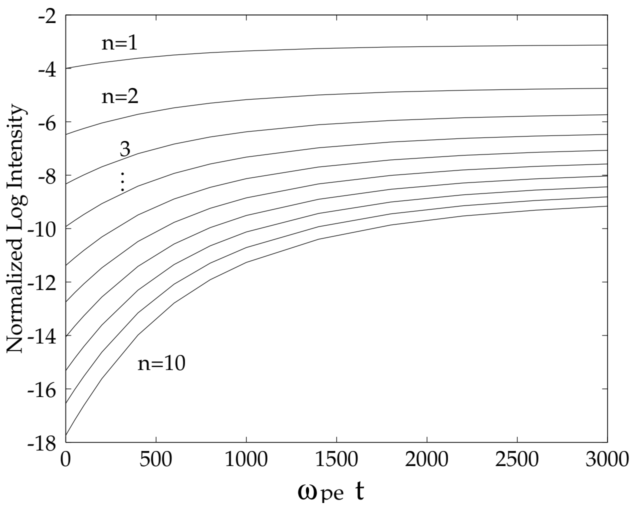

Fig. 2. Time evolution of the normalized wave intensity, max $I^+_{Ln}(t)/(8\pi\hat{n}T_e)$, versus $\omega_{pe}t$, in logarithmic vertical scale, for the first ten harmonics, obtained by solving theoretical Eqs. (8) and (9).

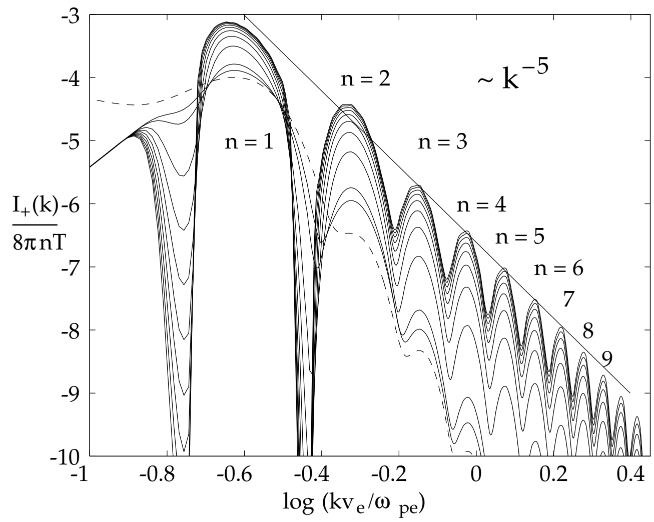

Fig. 3. Evolution of the total wavenumber spectrum, $I_+(k)/(8\pi\hat{n}T_e) = \sum_{n=1}^{10} I^+_{L_n}(k)/(8\pi\hat{n}T_e)$, versus kv_e/ω_{pe} (in log-log scale), computed on the basis of theoretical Eqs. (8) and (9), showing up to 10 harmonics.

form is not crucial in this regard. The quantity I_n is determined by the expression

$$I_n = I_1\, e^{-\beta(n-1)}\, n^{-\alpha}, \tag{11}$$

where I_1, α, and β are all constants that can be arbitrarily chosen. It turns out that this particular profile allows for a combination of exponential and power-law dependence among the peaks of saturated harmonic mode spectra. Since the dynamical evolution of the harmonic modes is dictated by quasilinear equation, the choice of input will be directly reflected in the saturation spectra. Our choice (11) gives us sufficient freedom to adjust our theory to match the simulation result to be shown later.

Figure 2 shows the time evolution for the peak of each harmonic mode spectrum, max $I^+_{Ln}(t)/(8\pi\hat{n}T_e)$. The parameters relevant to determine the initial spectra are $I_1 = 10^{-3}$, $\alpha = 5$ and $\beta = 2.236$. Note that the higher the harmonic mode number, the faster the mode grows initially, in agreement with the simulations (Klimas, 1983, 1990). Note also that all eigenmodes reach saturation at about the same time.

The evolution of the wave-number spectrum for weak harmonic Langmuir turbulence from the linear phase until quasi-saturation stage can be seen in Fig. 3. In this figure, where we have plotted the superposition of all the harmonic wave intensities,

$$I^+(k) = \sum_{n=1,2,3,\ldots} I^+_{Ln}(k),$$

the total time interval ranges from $\omega_{pe}t = 0$ to $\omega_{pe}t = 3400$. Note that the initial form of superposed spectra (dashed line) is not in the power-law form, but it achieves a power-law spectral shape at the saturation stage. This is owing to the fact that the higher harmonics grow faster than the lower harmonics. Our choice of spectral shape parameters, α and β, and the specific form of initial spectra were partly designed to produce a power-law form at quasi-saturation stage. If we connect the peaks of the individual harmonics, then one obtains an overall power-law,

$$I^+(k) \propto k^{-5}.$$

The spectrum shown in Fig. 3 bears a qualitative resemblance with some measurements made on weak beam-plasma systems (see, for instance, Fig. 4 of the paper by Apel (1969)). The spectral property in terms of frequency, instead of wave number, follows the same power-law pattern and is not shown here.

Since the results presented here practically correspond to the quasilinear stage of the kinetic evolution, the electron distribution function is mostly affected by the linear wave-particle interaction with the combined fundamental and nonlinear harmonic Langmuir modes. However, the energy content of the harmonic modes is very low compared to the fundamental mode. As a consequence, the temporal evolution of the distribution function is very similar to the customary quasilinear theory which does not include the harmonics.

To compare the theoretical spectrum with the simulation, we present the simulated spectrum in Fig. 4, where the normalized wave intensity near the saturation is plotted versus the wave number, in log-log scale. The power-law index of ~ -5 to -6 associated with the initial noise spectrum is indicated by the diagonal line. The wave spectrum shown in the figure corresponds to the near-saturation time of $\omega_{pe}t = 819.2$. As the readers can appreciate, the simulation shows that the final wave intensity features a quasi power-law-like spectra with an index of roughly between -5 and -6. From this, it appears that the turbulence spectrum of ~ -5 to -6 seems to be some sort of quasi universal constant, characterizing the Langmuir turbulence.

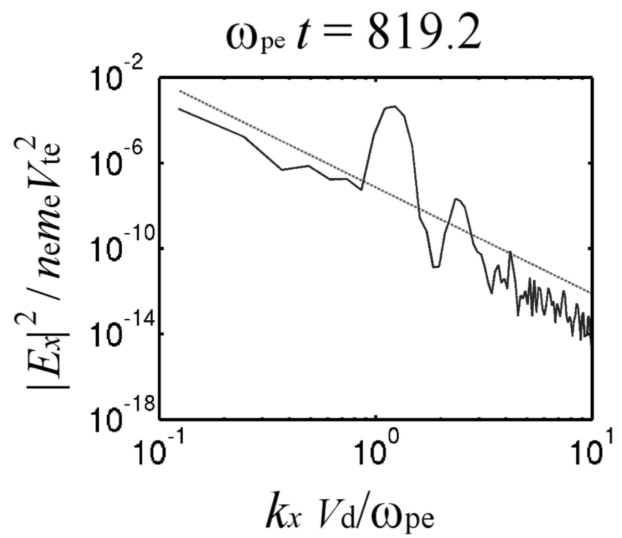

Fig. 4. Simulated spectrum at quasi-saturation. The straight diagonal line represents the power-law intensity distribution, $|E_x|^2(k_x)\propto k_x^{-5}$.

6 Conlusions and discussion

In this paper, we have briefly reviewed the history of Langmuir wave turbulence as developed over the past four decades or so. We have then presented a new twist on the Langmuir turbulence scenario which involves the generation of harmonics of Langmuir waves, which is essentially a quasilinear process, although the existence of the harmonic modes themselves cannot be discussed on the basis of linear theory. These modes are shown to form a quasi power-law distribution during the beam plateau formation stage, and saturate early. Then, the fully nonlinear wave-coupling processes, such as the well-known nonlinear decay and scattering processes, should proceed. As is well known, the combined effects of decay and scattering leads to the condensation of Langmuir waves in the long wavelength regime, over a time scale much longer than the quasilinear time scale of harmonic mode and power-law spectrum generation. The harmonic mode excitation thus seems to be capable of accounting for the quasi power-law wave spectrum within the context of weak turbulence theory.

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